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Interstrip resistance measurement

Technical Note

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In ATLAS SCT community two methods of interstrip resistance measurements are used: a) measuring the resistance between two strips and comparing it with a separately measured strip-to bias-rail resistance and b) applying DC voltage to one strip and measuring the current flowing to another strip. The method a) will further be referred to as Resistance Method and method b) as Induced Current Method.

1. Basic relations

Consider a semi-infinite chain of bias, R_b, and interstrip, R_{is}, resistors as shown in Fig.1. Assuming that all R_b are equal and the same is true for R_{is} one can find an equivalent resistance R_{eq} of the chain presented in Fig.1.

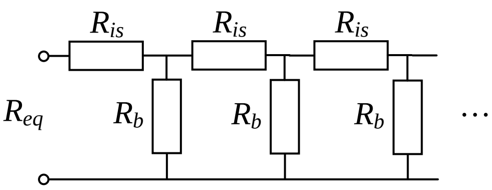

Fig.1. Circuit diagram for R_{eq}

As shown in Appendix A $R_{eq}=bR_b$ where

$$b = \frac{x + \sqrt{x^2 + 4x}}{2}$$

and $x=R_{is}/R_b$ is the parameter quantifying the interstrip isolation. Obviously $b>x$. For $x\to 0$ $b \to \sqrt{x}$ and $R_{eq} \to \sqrt{R_b R_{is}}$. For $x\to\infty$ $b\to x$ and $R_{eq}\to R_{is}$.

In both methods it is necessary to measure R_0 - the resistance between an individual strip and the bias rail. As demonstrated in Appendix A

$$R_0 = R_b \frac{b}{b+2} < R_b$$

When $x\to 0$ $R_0 \to \frac{\sqrt{R_b R_{is}}}{2} = \frac{R_{eq}}{2}$, while for $x\to\infty$ (b→∞) $R_0\to R_b$.

2. Resistance Method

Typically the resistance is measured between two adjacent strips. However in some situations the access is possible only to every second strip. Therefore this case is also considered.

2.1 Neighbour strips

The resistance R_1 between two adjacent strips can be expressed as (see Appendix B)

$$R_1 = R_{is}\frac{2}{b+2} < R_{is}$$

For $x\to 0$ ($b\to 0$) $R_1\to R_{is}$ while for $x\to\infty$ ($b\to x$) $R_1\to 2R_{is}/x=2R_b$. Note an interesting relation

$$\frac{R_0}{R_b}+\frac{R_1}{R_{is}}=1$$

The experimentally measured parameters R_0 and R_1 allow finding R_b and R_{is}. It is useful to introduce parameter $\rho_1=2R_0/R_1$. Then as shown in Appendix B

$$\rho_1 = \frac{b}{x} > 1; x = \frac{1}{\rho_1(\rho_1 - 1)}$$

$$R_b = R_0(2\rho_1 - 1)$$

$$R_{is} = R_1\frac{2\rho_1 - 1}{2(\rho_1 - 1)}$$

When $x\to\infty$ ($b\to x$) $\rho_1\to 1$, $R_b\to R_0$ and $R_{is}\to\infty$. In this situation $\varepsilon_1=\rho_1$-1 is close to $1/x$. It is the experimentally achievable accuracy in ε_1 that limits the maximum reliably measurable R_{is}. If e.g. the minimum reliably measurable ε_1 is estimated to be 0.05 then the maximum measurable $x=R_{is}/R_b$ is ~20.

2.2 Next neighbour strips

As in the previous section introduce $\rho_2=2R_0/R_2$ where R_2 is the resistance between two next neighbour strips. As shown in Appendix B

$$\rho_2 = \frac{(b+1)^2}{b(b+2)} > 1; x = \frac{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2}{\sqrt{\rho_2(\rho_2 - 1)}}$$

$$R_b = R_0\left(\sqrt{\rho_2} + \sqrt{(\rho_2 - 1)}\right)^2$$

$$R_{is} = \frac{R_2}{2}\sqrt{\frac{\rho_2}{\rho_2 - 1}}$$

When $x \to \infty$ ($b \to x$) $\rho_2 \to 1$, $R_b \to R_0$ and $R_{is} \to \infty$. In this situation $\varepsilon_2 = \rho_2 - 1$ is close to $1/x^2$. Again the accuracy in ε_2 limits the maximum reliably measurable R_{is}. For minimum reliably measurable $\varepsilon_2 = 0.05$ the maximum measurable $x = R_{is}/R_b$ is 2.8 i.e. ~7 times smaller than for the same accuracy in ε_1. Thus the R_{is} reconstruction ability for measurements with next neighbours is significantly worse than that for the adjacent strips.

Fig.2 shows the ε_1 and ε_2 as a function of x. The lines are $1/x$ and $1/x^2$ dependences.

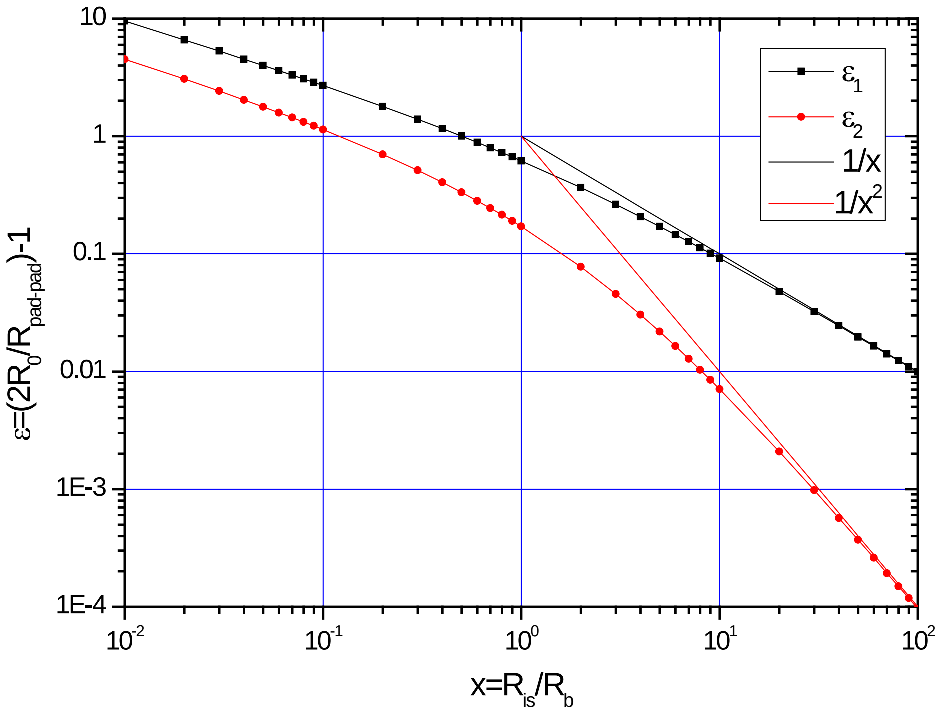

Fig.2. Deviation of ρ_1 and ρ_2 from unity vs. x (see text for further details)

Sometimes a more pragmatic approach is used when only “pad-pad” conductivity is measured as a function of bias and the voltage, above which this conductivity exceeds its plateau value by less than some fraction (e.g. δ<10%), is considered to be the “strip isolation” voltage. Note that without any additional information this approach doesn’t tell quantitatively how good the isolation is. If a usual assumption is made that the plateau value is equal to $1/(2R_0)$ then for the adjacent strips δ coincides with ε_1 and the plot in Fig.2 (or corresponding formula) allow an estimate of the lower limit for x

(e.g. for δ<0.1 x>9). The pragmatism here is in assuming the pad-pad conductivity plateau value to be one half of the conductivity between the pad and the bias rail without actually measuring the latter.

3. Induced Current Method

When potential V_0 is applied between a strip and the bias rail it induces a current flowing via R_{is} to the neighbouring strips. This current can be measured either directly or indirectly by connecting an ammeter or voltmeter parallel to the bias resistor, R_b, of the investigated strip. To work well the first approach requires the ammeter internal resistance to be much smaller than R_b while the second one requires the voltmeter internal resistance to be much larger than R_b. The latter condition can be easily satisfied because for modern voltmeters R_{int}~10GΩ. For the ammeters however an ability to measure the currents below nA level is usually accompanied by an internal resistance being of an order of MΩ. Thus the applicability range of the current measurement approach is narrower than that of the voltage one and because of this the former will not be considered further in this Note.

3.1 Neighbour strips

Typical approach is to connect between a strip and the bias rail a source-meter unit (SMU) providing potential V_0 varying by a few volts around zero and to measure the current flowing out of the SMU, which allows R_0 measurement. Simultaneously a voltmeter connected between a neighbour strip and the bias rail measures the potential V_1 as a function of V_0. Ideally the induced potential V_1 should be simply proportional to V_0 and their ratio would characterise the inter-strip isolation. In practice however the slope $S_1=dV_1/dV_0$ is used instead of V_1/V_0.

As shown in Appendix C the slope dependence on x is very simply expressed via b:

$$S_1 = \frac{1}{1+b}$$

For $x\to 0$ ($b\to 0$) $S_1\to 1$ while for $x\to\infty$ ($b\to x$) $S_1\to 1/x\to 0$. Experimentally measured parameters R_0 and S_1 allow reconstruction of the parameters in question: R_b and R_{is}. As demonstrated in Appendix C

$$R_b = R_0 \frac{1+S_1}{1-S_1}$$

$$R_{is} = R_0 \left(\frac{1}{S_1} - S_1 \right).$$

In a typical situation when $x \to \infty$, $S_1 \to 0$ one gets $R_b \to R_0$ and $R_{is} \to R_0/S_1 = R_0(dV_0/dV_1)$. A minimum detectable slope S_1 defines the maximum measurable R_{is}. For a proven to be detectable S_1 of ~10^{-6} the limit for R_{is} is ~$10^6 R_0$, which for a typical R_0~1MΩ corresponds to R_{is} ~1000GΩ.

3.2 Next neighbour strips

As shown in Appendix C the slope of the voltage induced on the next neighbour strip $S_2 = dV_2/dV_0$ is related to S_1 in a very simple way: $S_2 = S_1^2$. Therefore the reconstruction formulae become

$$R_b = R_0 \frac{1+\sqrt{S_2}}{1-\sqrt{S_2}}$$

$$R_{is} = R_0 \left(\frac{1}{\sqrt{S_2}} - \sqrt{S_2} \right).$$

For $x \to \infty$ $S_2 \to 0$ as $1/x^2$. Therefore for the same limit of measurable S_2 ~10^{-6} the limit for the R_{is} is ~$10^3 R_0$, which for a typical R_0~1MΩ corresponds to R_{is} ~1GΩ.

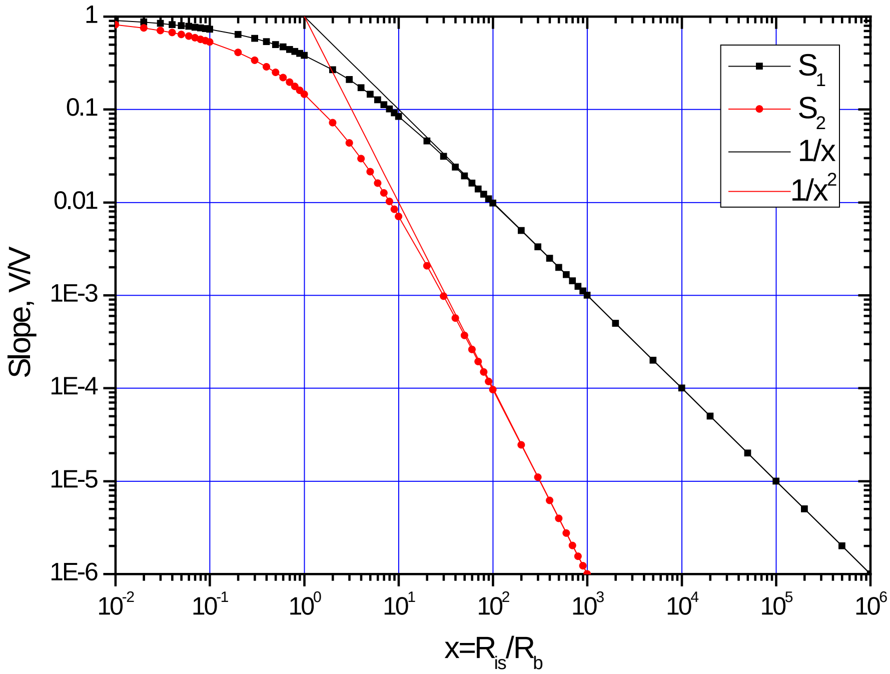

Fig.3. S_1 and S_2 vs.x.

Fig.3 shows S_1 and S_2 as a function of x together with $1/x$ and $1/x^2$ lines. Similarly to the Resistance Method the measurements with neighbour strip have higher sensitivity compared to that for the next neighbour strip. But even for the latter the sensitivity is much higher than what can be achieved by the Resistance Method.

3.3 The effects of non-zero resistance to the ground

Another limit for the maximum measurable R_{is} appears from a non-zero resistance to the ground R_g. As demonstrated in Appendix C in a typical situation $R_g << R_b << R_{is}$ the R_{is} calculated from the measured S_1 represents the actual R_{is} in parallel with the effective parasitic resistance $R_p = R_b^2/R_g$. For a typical R_0~1MΩ even R_g~1Ω results in R_p~1000GΩ, which sets a practical limit for the sensitivity of the induced voltage method.

As explained in Appendix C a non-zero R_g results in an offset to the slope, which is the same for the neighbour and the next neighbour strips. Comparison of S_1 and S_2 measured under the same conditions allows decoupling of the effects related to R_{is} and R_g. An example of such analysis is presented in Fig.4.

The measurements were performed at room temperature and ~40% relative humidity with non-irradiated *n*-in-*p* microstrip sensor w27-bz1-p7 produced by Hamamatsu within the ATLAS Tracker Upgrade R&D Program. The sensor has 104 strips with a pitch of 74.5 μm and a length of 8 mm. There is no p-spray or p-stop interstrip isolation in this sensor, which results in a relatively low R_{is} values even at quite high bias values. The sensor depletion voltage is 153V. The bias voltage was changing downwards from 300V after the sensor was kept at this bias for ~3 hours. Three consecutive strips 59, 60 and 61 were used. At each bias value two separate V_0 scans were performed with the SMU connected either to strip 60 or 59 (with the connection to the other of these two strips floating) and the potential induced at the strip 61 was measured. In this way both slopes S_1 and S_2 were measured for each bias point.

Fig.4a shows S_1 and S_2 as a function of bias voltage. As expected S_2 is usually lower than S_1 but at $U_{bias} \geq 200V$ both slopes are very close and do not change with bias. This indicates that in this bias range both slopes are dominated by the R_g contribution. For comparison S_1^2 is also shown outside the high bias range. This curve is in a reasonable agreement with S_2.

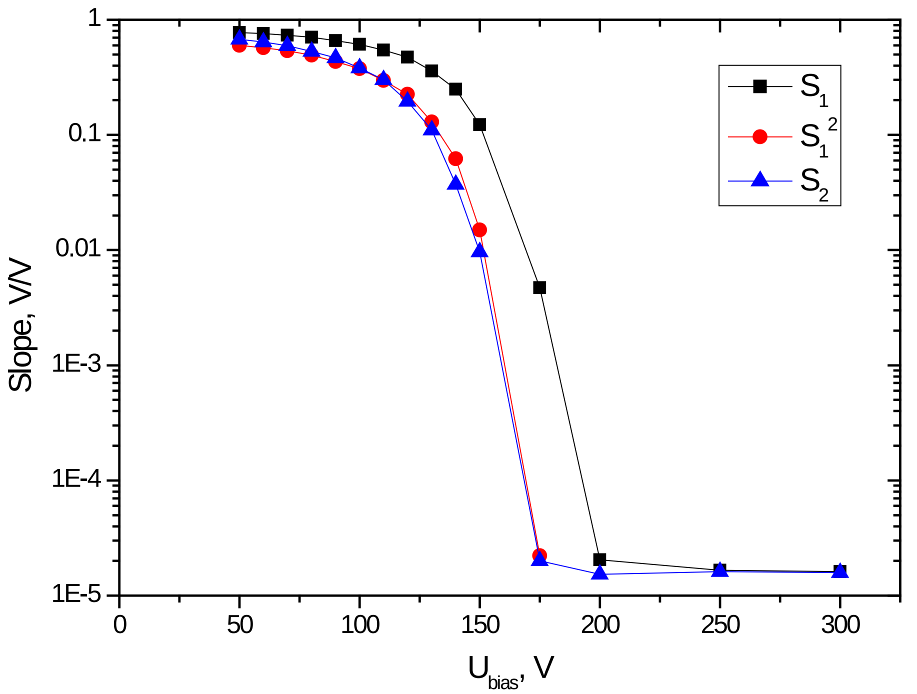

Fig.4a. Bias dependence of S_1 and S_2

The constant slope level observed at high bias was subtracted from S_1, S_2 and the R_{is} and R_b were calculated using these corrected slopes. The results are presented in Figs.4b and 4c respectively.

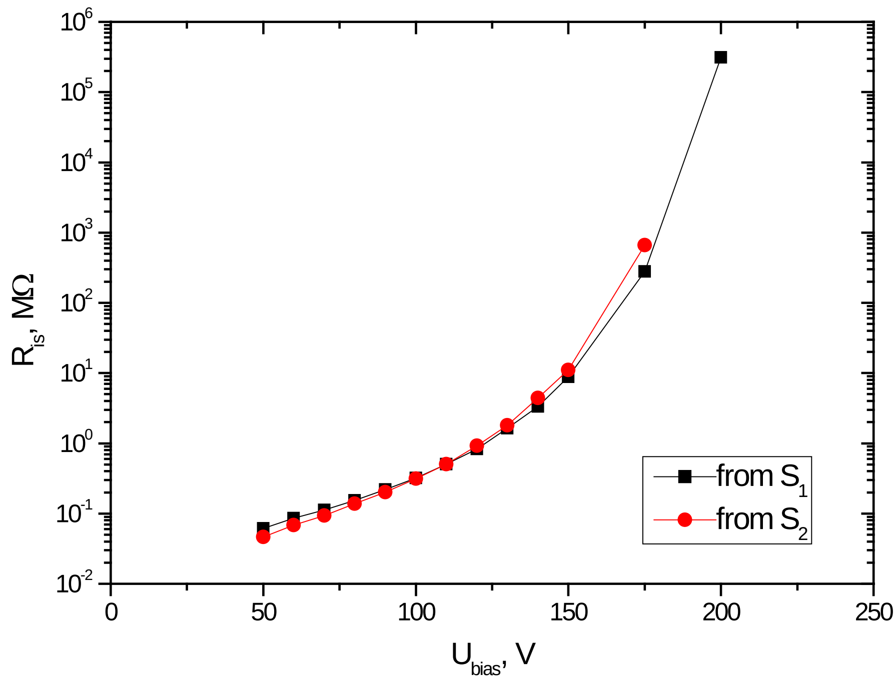

Fig.4b. The interstrip resistance calculated from the corrected S_1 and S_2

The R_{is} reconstructed from S_1 and S_2 agree quite well. Better sensitivity of the measurement with the neighbour strip allows a wider bias range of measurable R_{is}.

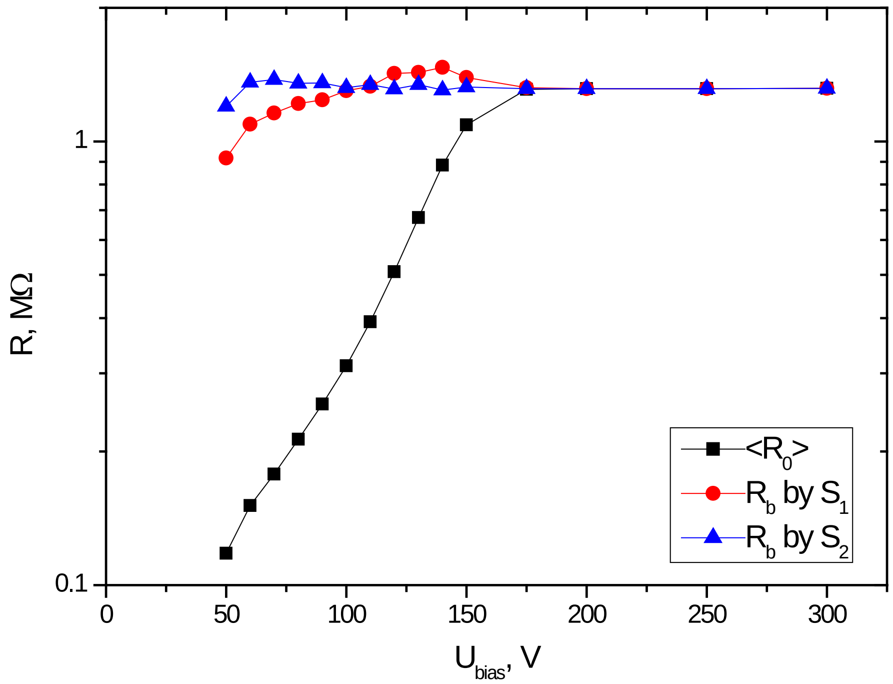

Fig.4c. Measured, R_0, and reconstructed, R_b, resistances

At low bias the measured resistance R_0 is significantly lower than its plateau level corresponding to the bias resistor value. However the reconstructed R_b remains approximately constant down to the lowest bias point. It is interesting that the bias resistance reconstruction works better for the next neighbour strip data.

Consistency between the results obtained from the measurements with neighbour and next neighbour strips validates the model used in the calculations.

Appendix A. Basic relations details

Looking at the circuit presented in Fig.1 one may notice that R_{eq} can be presented as R_{is} plus R_b parallel to R_{eq} that results in the following equation*

$$R_{eq} = R_{is} + \frac{R_b R_{eq}}{R_b + R_{eq}} \quad \text{(A.1)}$$

Using parameters $x=R_{is}/R_b$ and $b=R_{eq}/R_b$ the eq. (A.1) can be re-written as

$$b = x + \frac{b}{1+b} \quad \text{(A.2)}$$

or

$$b^2 - xb - x = 0 \quad \text{(A.3)}$$

from which it follows:

$$x = \frac{b^2}{1+b} \quad \text{(A.4)}$$

and

$$b = \frac{x + \sqrt{x^2 + 4x}}{2}. \quad \text{(A.5)}$$

The last is the result of solving eq. (A.3) vs. b and keeping only the positive solution.

The resistance R_0 between a strip and the bias rail consists of three resistors in parallel: bias resistor R_b and two R_{eq} i.e.

$$\frac{1}{R_0} = \frac{1}{R_b} + \frac{2}{R_{eq}} \quad \text{(A.6)}$$

Using $R_{eq}=bR_b$ one gets from eq. (A.6)

$$R_0 = R_b \frac{b}{b+2} \quad \text{(A.7)}$$

Appendix B. Resistance method calculations

a) Adjacent strips

An equivalent circuit diagram for measuring resistance R_1 between two adjacent strips is shown in Fig.B1. R_1 is the resistance between the points A and B and can be expressed as follows

$$\frac{1}{R_1} = \frac{1}{R_{is}} + \frac{R_b + R_{eq}}{2R_b R_{eq}} \quad \text{(B.1)}$$

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* I am indebted to Nobu Unno for the idea of this calculation. – A.C.

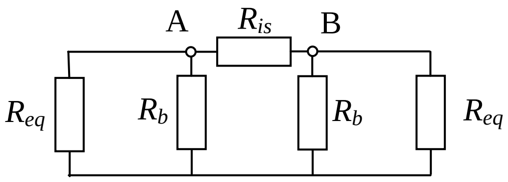

Fig.B1. Equivalent circuit diagram for measuring R_1

Using parameters $x=R_{is}/R_b$ and $b=R_{eq}/R_b$ the eq. (B.1) can be re-written as

$$\frac{R_{is}}{R_1} = 1 + \frac{x(b+1)}{2b} \qquad \text{(B.2)}$$

Expressing x via b using eq.(A.4) one gets from (B.2)

$$R_1 = R_{is}\frac{2}{b+2} \qquad \text{(B.3)}$$

Introduce parameter $\rho_1=2R_0/R_1$. Using eqs. (A.7) and (B.3) one obtains

$$\rho_1 = \frac{bR_b}{R_{is}} = \frac{b}{x} \qquad \text{(B.4)}$$

Expressing b via x from eq.(A.5) and finding x from the resulting equation one gets

$$x = \frac{1}{\rho_1(\rho_1 - 1)} \qquad \text{(B.5)}$$

Combining eqs. (B.4) and (B.5) one can express b vs. ρ_1:

$$b = \frac{1}{\rho_1 - 1} \qquad \text{(B.6)}$$

Using eq. (B.6) one obtains from (A.7)

$$R_b = R_0(2\rho_1 - 1) \qquad \text{(B.7)}$$

and from (B.3)

$$R_{is} = R_1\frac{2\rho_1 - 1}{2(\rho_1 - 1)} \qquad \text{(B.8)}$$

b) Next-neighbour strips

An equivalent circuit diagram for measuring resistance R_2 between two next-neighbour strips is shown in Fig.B2. Due to the symmetry there is no potential difference between the ends of the central bias resistor. Therefore it can be either removed or replaced by a short.

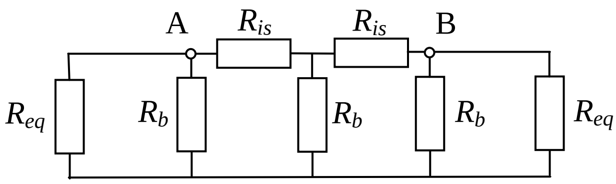

Fig.B2. Equivalent circuit diagram for measuring R_2

In both cases the resistance R_2 between the points A and B can be expressed as:

$$\frac{1}{R_2} = \frac{1}{2}\left(\frac{1}{R_{is}} + \frac{1}{R_b} + \frac{1}{R_{eq}}\right) \qquad \text{(B.9)}$$

From (B.9) the parameter $\rho_2=2R_0/R_2$ can be expressed as (using also eq. (A.7)):

$$\rho_2 = \frac{R_0}{R_b}\left(1 + \frac{R_b}{R_{is}} + \frac{R_b}{R_{eq}}\right) = \frac{b}{b+2}\left(1 + \frac{1}{x} + \frac{1}{b}\right) \qquad \text{(B.10)}$$

Using for x its form of eq. (A.4) one finally obtains:

$$\rho_2 = \frac{(b+1)^2}{b(b+2)} \qquad \text{(B.11)}$$

As follows from (B.11)

$$\rho_2 - 1 = \frac{1}{b(b+2)}; \frac{\rho_2}{\rho_2 - 1} = (b+1)^2; b+1 = \frac{\sqrt{\rho_2}}{\sqrt{\rho_2 - 1}} \qquad \text{(B.12)}$$

The last part can be transformed into

$$b^2 = \frac{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2}{\rho_2 - 1} \qquad \text{(B.13)}$$

Substituting in eq. (A.4) (b+1) from (B.12) and b^2 from (B.13) one obtains

$$x = \frac{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2}{\sqrt{\rho_2(\rho_2 - 1)}} \qquad \text{(B.14)}$$

To express R_b via R_0 and ρ_2 one can re-write eq. (A.7)

$$R_b = R_0\frac{b+2}{b} = R_0\frac{b(b+2)}{b^2} \qquad \text{(B.15)}$$

Substituting $b(b+2)$ by $1/(\rho_2-1)$ from eq. (B.12) and using b^2 from (B.13) one gets

$$R_b = \frac{R_0}{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2} = R_0\left(\sqrt{\rho_2} + \sqrt{\rho_2 - 1}\right)^2 \qquad \text{(B.16)}$$

To find R_{is} one can transform (B.16) as follows

$$R_{is} = xR_b = \frac{xR_0}{\left(\sqrt{\rho_2} - \sqrt{\rho_2 - 1}\right)^2} \tag{B.17}$$

Using the relation $R_0=(\rho_2 R_2)/2$ following from the ρ_2 definition and x from (B.14) one obtains from (B.17)

$$R_{is} = \frac{R_2}{2}\sqrt{\frac{\rho_2}{\rho_2 - 1}} \tag{B.18}$$

Appendix C. Induced voltage calculations

An equivalent circuit diagram for the R_{is} measurement using the voltage induced at the neighbour strip is presented in Fig.C1. The SMU is connected between the point marked V_0 and the ground while the induced voltage is measured between the point marked V_1 and the ground.

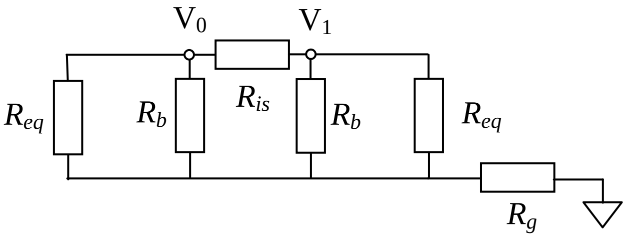

Fig.C1. Equivalent circuit diagram for measurement with the neighbour strip

First let us consider a situation when the resistance to the ground, R_g, is zero.

a) Zero R_g.

As follows from the diagram in Fig.C1 the induced voltage V1 can be expressed via applied voltage V0 as

$$V_1 = V_0 \frac{\dfrac{R_b R_{eq}}{R_b + R_{eq}}}{R_{is} + \dfrac{R_b R_{eq}}{R_b + R_{eq}}} = V_0 \frac{R_b R_{eq}}{R_{is}\left(R_b + R_{eq}\right) + R_b R_{eq}} \tag{C.1}$$

Using the relations $R_{is}=xR_b$, $R_{eq}=bR_b$ and the eq. (A.4) one finds from (C.1)

$$S_1 = \frac{V_1}{V_0} = \frac{1}{1+b} \tag{C.2}$$

Expressing from (C.2) b via S_1 one gets

$$b = \frac{1 - S_1}{S_1}. \qquad \text{(C.3)}$$

Substituting *b* in eq. (A.7) by its expression from (C.3) one obtains

$$R_b = R_0 \frac{1 + S_1}{1 - S_1}. \qquad \text{(C.4)}$$

Substituting *b* in eq. (A.4) by its expression from (C.3) one obtains

$$x = \frac{(1 - S_1)^2}{S_1}. \qquad \text{(C.5)}$$

Using the relation $R_{is}=xR_b$ and eqs. (C.4), (C.5) one gets

$$R_{is} = R_0 \left(\frac{1}{S_1} - S_1 \right). \qquad \text{(C.6)}$$

Typically the parameter R_0 is measured with a good accuracy while S_1 (especially when it is very small) has a significant relative error $\sigma(S_1)/S_1$. Using (C.6) one can calculate the uncertainty in R_{is} due to the error in S_1

$$\sigma(R_{is}) = R_0 \frac{\sigma(S_1)}{S_1} \left(\frac{1}{S_1} + S_1 \right). \qquad \text{(C.7)}$$

For measurement with the next neighbour strip the diagram shown in Fig.C1 can also be used but using potential V_1 instead of V_0 and V_2 instead of V_1. Obviously one gets in this case

$$\frac{V_2}{V_1} = \frac{V_1}{V_0} \rightarrow S_2 = \frac{V_2}{V_0} = \left(\frac{V_1}{V_0} \right)^2 = S_1^2 \rightarrow S_1 = \sqrt{S_2}. \qquad \text{(C.8)}$$

Therefore for the measurements with next neighbour strips the eqs. (C.4) and (C.6) can be written as

$$R_b = R_0 \frac{1 + \sqrt{S_2}}{1 - \sqrt{S_2}} \qquad \text{(C.9)}$$

$$R_{is} = R_0 \left(\frac{1}{\sqrt{S_2}} - \sqrt{S_2} \right) \qquad \text{(C.10)}$$

while the eq. (C.7) is transformed to

$$\sigma(R_{is}) = \frac{R_0}{2} \frac{\sigma(S_2)}{S_2} \left(\frac{1}{\sqrt{S_2}} + \sqrt{S_2} \right). \qquad \text{(C.11)}$$

b) Effects of non-zero R_g.

As can be seen from the circuit diagram presented in Fig.C1 a non-zero resistor R_g results in a voltage drop on it

$$V_g = V_0 \frac{R_g}{R_0 + R_g}, \qquad \text{(C.12)}$$

which adds up to V_1 or V_2 that would be measured with R_g=0. In other words the measured slopes S_1, S_2 will include an additional component

$$S_g = \frac{V_g}{V_0} = \frac{R_g}{R_0 + R_g}, \qquad \text{(C.13)}$$

which is the same for neighbour and the next neighbour strips. The S_g may be measured e.g. as the slope in the situation S_1=S_2. It then can be subtracted from the slopes measured under other conditions thus suppressing the effects from non-zero R_g.

To verify this model a special measurement was made with 1 kΩ resistor inserted between the bias rail and the ground. The measurements were performed with an SCT End-Cap sensor w31-225 at U_{bias}=50 V. Fig.C2 summarises the results.

With grounded bias rail the slope dV_1/dV_0 of the voltage induced at the neighbour strip was found to be 4.7±0.2 μV/V, as can be seen from the experimental data in Fig.C2a.

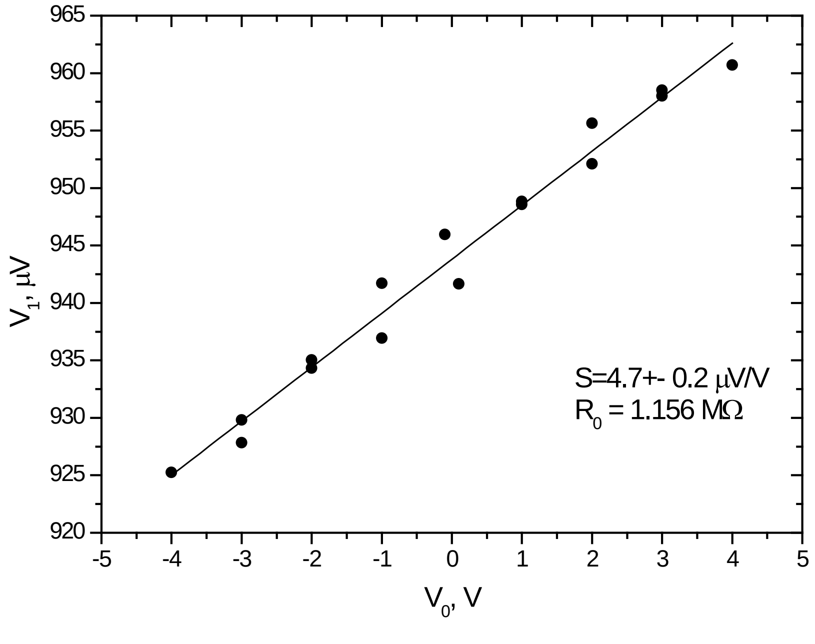

Fig.C2a. V_1 vs. V_0 for grounded bias rail

When 1.0 kΩ resistor was inserted between the bias rail and the ground, the slope increased to 869±5 μV/V as shown in Fig.C2b.

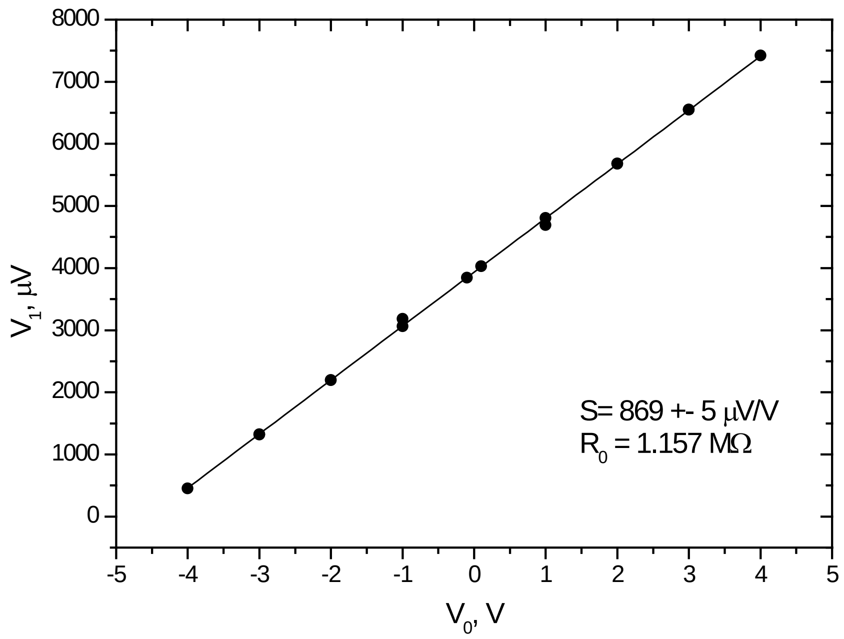

Fig.C2b. V_1 vs. V_0 for 1 kΩ resistor between the bias rail and the ground

In the first case R_0 was measured to be 1156 kΩ while in the second it increased to 1157 kΩ due to the additional 1 kΩ resistor. The slope S_1 due to 1 kΩ resistor calculated from the eq. (C.13) is S_g=1kΩ/1157kΩ=864.3 10^{-6}=864.3 μV/V. Adding it to the initial slope S_1=4.7 μV/V one obtains for the second measurement 869 μV/V in a perfect agreement with the results presented in Fig.C2b that validates the model.

For further discussion let's restrict ourselves to a typical in practice situation $R_g << R_0$. Then the usual gradient dV_0/dI_0 still correctly measures R_0 and the additional slope $S_g=R_g/R_0<<1$. Let us now consider only the situation when the real slopes S_1, S_2 are comparable with S_g and therefore the effects of R_g are essential. The measured slopes S_1^{meas}, S_2^{meas} are then also << 1. For the measurements with the neighbour strip we then obtain from eq. (C.6) for the measured interstrip resistance

$$\frac{1}{R_{is}^{meas}} = \frac{1}{R_0} S_1^{meas} = \frac{1}{R_0}\left(S_1 + S_g\right) = \frac{1}{R_{is}} + \frac{R_g}{R_0^2}. \qquad \text{(C.14)}$$

As follows from (C.14) the measured R_{is} is equal to the actual R_{is} with an effective parasitic resistance $R_p=R_0^2/R_g$ connected in parallel and restricting the measurable R_{is}. For typical values of R_0=1MΩ and R_g=1Ω R_p=1000GΩ.

For the measurements with next neighbour strip one can obtain from eq. (C.10)

$$\frac{1}{(R_{is}^{meas})^2} = \frac{1}{R_0^2} S_2^{meas} = \frac{1}{R_0^2}\left(S_2 + S_g\right) = \frac{1}{R_{is}^2} + \frac{R_g}{R_0^3}. \qquad \text{(C.15)}$$

Thus for the next neighbour the measured R_{is} is limited by an effective parasitic resistance $R_p = R_0\sqrt{\frac{R_0}{R_g}}$. For typical values of R_0=1MΩ and R_g=1Ω R_p=1GΩ.